

Small Radius Perturbation of the Selfgravitating Gas with Cylindrical Symmetry *

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1 Abstract

Self-consistent movement of initial perturbation in density, velocity and gravitation potential on the background of the stationary cylindrical configuration of the gas with gravitation and pressure in Lagrange variables have been studied. The nonlinear partial differential equation for description radius motion has been obtained. The linearization of this equation is reduced to a Klain-Gordon equation which has an analytical solution.

2 The Model

Self-consistent movement of initial perturbation in density, velocity and gravitation potential on the background of the stationary cylindrical configuration of the gas with gravitation and pressure in Lagrange variables have been studied. In [1] it is shown that a cylindrical symmetry admits an equilibrium state of the gas with selfgravitating and pressure. The cylinder is supposed infinity long along the axes and all physical values are dependent on the radius of cylinder only. The equilibrium is provided by the equality of gravitational force and force of gas pressure in every point. In this article the model used in [1] is appeared as a background for a small perturbations in velocity, density, pressure and gravitaitational potential. In general case the perturbation may depend from two coordinats and have very complicated geometrical form. It is not possible to study two-dimensional nonlinear motion in general case. But the small deviation from the stationary state admits the principle of superposition. According to this it is possible to study radius and longitudinal motions independently

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and then to sum them taking into consideration the fact that they are vectors. This article is dedicated to study pure radius motion.

The model of perturbation depending on radius of cylinder only is choosen. It means that the cylinder infinite along the axes of symmetry makes (as a whole) radius motion by the influence of initial perturbation of stationary state. The perturbation is distributed along the axes of cylinder thus its value is dependent on the radius coordinat only. Two causes are able to generate the initial perturbation: 1) the velocity is not equal to zero for particles in the equilibrium state or 2) displacement of a particle from the equilibrium point is not equal to zero. The combination of the both is possible as well.

The mathematical description of this model is represented by the Cauchy problem for three-dimensional nonstationary partial differential equations of motion, continuity, Poisson equation and algebraic equation of state:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{\nabla P}{\rho} - \nabla \Phi \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \quad (2)$$

$$\nabla^2 \Phi = 4\pi G \rho \quad (3)$$

$$P = A \rho^\gamma \quad (4)$$

whis initial conditions

$$\vec{v}|_{t=0} = \vec{v}(\vec{r}, 0), \quad P|_{t=0} = P(\vec{r}, 0), \quad \Phi|_{t=0} = \Phi(\vec{r}, 0), \quad \rho|_{t=0} = \rho(\vec{r}, 0) \quad (5)$$

where P is pressure, \vec{v} - speed, Φ - gravitational potential, ρ - gas density, t - time, $A = const$, $1 \leq \gamma \leq 2$. $\vec{v}(\vec{r}, 0)$, $P(\vec{r}, 0)$, $\Phi(\vec{r}, 0)$ and $\rho(\vec{r}, 0)$ are specified functions. The system (1) - (5) describes hydrodynamic motion in the ideal classic gas with selfgravitation and pressure.

3 Characteristic Values and Parameters

Characteristic values are needed to transfer from dimension to dimensionless equations. A choise of characteristic values is defindent on the physical model. Having been choosen these values constitute dimensionless coefficients of the equations. Depending on the initial and/or boundary conditions these coefficients are able to play a role of small or big parameters. There are two kinds of characteristic dimensional values in this problem. One kind of them is connected with stationary gas configuration (the background) and the second one - with perturbations expanded on the stationary background. Some various perturbations dependent on the initial conditions have distinction by different time characteristics (a period, for example). To compare them it is essentially to

have a time scale not dependent on the perturbation. There is only one choice to do so - to attach the time scale to the background.

The stationary configuration of the gas will be described by a series of the characteristic values - Jeans length L_0 , gravitational potential Φ_0 , gas pressure P_0 , mass of particle m , concentration of particles N_0 , density ρ_0 , temperature T_0 , sound speed c_0 :

$$L_0^2 = \frac{\pi c_0^2}{G \rho_0} \quad \Phi_0 = \frac{Gm}{L_0} \quad P_0 = N_0 k T_0 \quad c_0^2 = A \gamma \rho_0^{\gamma-1} \quad (6)$$

that is why:

$$t_{scale} = \frac{1}{\sqrt{G \rho_0}}. \quad (7)$$

Full energy of the gas consists of gravitational energy and energy of the gas thermal expansion. The full energy is distributed between these two components depending on the adiabatic index γ . The function $\mu(\gamma)$ describes this distribution as follows:

$$\mu(\gamma) = \frac{k T_0}{m \Phi_0}. \quad (8)$$

Equations (6) - (8) give the correlation between a number of particles in the cube with side length equal to Jeans length and parameter $\mu(\gamma)$:

$$N_0 L_0^3 = \pi \gamma \mu(\gamma). \quad (9)$$

The perturbation will be characterized by the following values defined by the initial conditions: the length of wave λ_0 , period of perturbation t_0 , velocity v_0 , characteristic Mach number and characteristic dimensionless wave length. They are:

$$\lambda_0 = v_0 t_0 \quad M_0 = \frac{v_0}{c_0} \quad \kappa_0 = \frac{\lambda_0}{L_0}. \quad (10)$$

A parameter α will also be used:

$$\alpha = \frac{t_0}{t_{scale}}. \quad (11)$$

From (6), (10) and (11) it follows:

$$\alpha = \frac{\sqrt{\pi} \kappa_0}{M_0}. \quad (12)$$

4 The Initial System of Equations in the Lagrangian Variables

To convert from dimensional equation (1) - (5) to dimensionless ones the dimensionless radius, density, pressure, gravitational potential and velocity are

introduced according to the following rules:

$$\xi = \frac{r}{L_0} \quad \delta = \frac{\rho}{\rho_0} \quad p = \frac{P}{P_0} \quad \varphi = \frac{\Phi}{\Phi_0} \quad v = \frac{v}{v_0}. \quad (13)$$

In the cylindrical system of coordinates the dimensionless radius component of equations (1) - (5) are

$$\frac{\partial v}{\partial \tau} + a_1 v \frac{\partial v}{\partial \tau} = -\frac{a_2}{\delta} \frac{\partial P}{\partial \xi} - a_3 \frac{\partial \varphi}{\partial \xi} \quad (14)$$

$$\frac{\partial \rho}{\partial \tau} + \frac{a_1}{\xi} \frac{\partial (\xi \delta v)}{\partial \xi} = 0 \quad (15)$$

$$\frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial \varphi}{\partial \xi} \right) = a_4 \delta \quad (16)$$

$$p = \delta^\gamma \quad (17)$$

with initial conditions

$$v|_{t=0} = v(\xi, 0) \quad p|_{t=0} = p(\xi, 0) \quad \phi|_{t=0} = \phi(\xi, 0) \quad \delta|_{t=0} = \delta(\xi, 0) \quad (18)$$

where $v(\xi, 0)$, $p(\xi, 0)$, $\phi(\xi, 0)$ and $\delta(\xi, 0)$ are specified functions and

$$a_1 = \frac{v_0 t_0}{L_0} \quad a_2 = \frac{P_0 t_0}{\rho_0 L_0 v_0} \quad a_3 = \frac{\Phi_0 t_0}{L_0 v_0} \quad a_4 = \frac{4\pi G \rho_0 L_0^2}{\Phi_0}. \quad (19)$$

According to the (6) and (13) the expressions for a_1 - a_4 become:

$$a_1 = \frac{\alpha M_0}{\sqrt{\pi}} \quad a_2 = \frac{\alpha M_0}{\gamma \sqrt{\pi}} \quad a_3 = \frac{\alpha}{\sqrt{\pi} \gamma \mu(\gamma) M_0} \quad a_4 = 4\pi^2 \gamma \mu(\gamma). \quad (20)$$

Finally in Euler variables the equations being looked for are obtained by excluding the α from the (20) and substituting the result into the (14) - (16):

$$\frac{1}{\kappa_0} \frac{\partial v}{\partial \tau} + v \frac{\partial v}{\partial \xi} = -\frac{1}{\gamma \delta} \frac{\partial P}{\partial \xi} - \frac{1}{\gamma \mu(\gamma) M_0^2} \frac{\partial \varphi}{\partial \xi} \quad (21)$$

$$\frac{\partial \delta}{\partial \tau} + \frac{\kappa_0}{\xi} \frac{\partial (\xi \delta v)}{\partial \xi} = 0 \quad (22)$$

$$\frac{1}{\xi} \frac{\partial}{\partial \xi} \left(\xi \frac{\partial \varphi}{\partial \xi} \right) = 4\pi^2 \gamma \mu(\gamma) \delta \quad (23)$$

with initial conditions (18). But the problem under consideration may be solved in the Lagrange coordinates only. Applying the transformation rules ([2]):

$$\frac{\partial}{\partial \tau} \rightarrow \frac{\partial}{\partial \tau} - \xi \delta v \frac{\partial}{\partial \sigma} \quad \frac{\partial}{\partial \xi} \rightarrow \xi \delta \frac{\partial}{\partial \sigma} \quad \text{where} \quad \sigma = \int_0^\xi \delta \xi d\xi \quad (24)$$

(σ is a dimensionless Lagrange mass variable) to equations (21) - (23) gives the system of equations describing the problem under consideration in Lagrangian system of variables:

$$\frac{1}{\kappa_0} \frac{\partial v}{\partial \tau} = -\frac{\xi}{\gamma} \frac{\partial p}{\partial \sigma} - \frac{\delta \xi}{\gamma \mu(\gamma) M_0^2} \frac{\partial \varphi}{\partial \sigma} \quad (25)$$

$$\frac{\partial \delta}{\partial \tau} + \kappa_0 \delta^2 \frac{\partial(v\xi)}{\partial \sigma} = 0 \quad (26)$$

$$\frac{\partial}{\partial \sigma} \left(\xi^2 \delta \frac{\partial \varphi}{\partial \sigma} \right) = 4\pi^2 \gamma \mu(\gamma) \quad (27)$$

with initial conditions:

$$\begin{aligned} v|_{t=0} = v(\sigma, 0) \quad p|_{t=0} = p(\sigma, 0) \quad \phi|_{t=0} = \phi(\sigma, 0) \quad \delta|_{t=0} = \delta(\sigma, 0) \\ \xi|_{t=0} = \xi(\sigma, 0) \quad \delta(0, 0) = 1 \end{aligned} \quad (28)$$

where $v(\sigma, 0)$, $p(\sigma, 0)$, $\phi(\sigma, 0)$, $\delta(\sigma, 0)$ and $\xi(\sigma, 0)$ are specified functions. The dependence of function $\xi(\sigma, 0)$ from σ will not be used in this article and, that is why, will not be studied.

5 The Development of the Radius Motion Equation

In this section the system of equations (25) - (27) will be reduced to one equation for Euler coordinate $\xi(\sigma, \tau)$. The dependence on (σ, τ) will be omitted except the case $\tau = 0$. It means that δ means $\delta(\sigma, \tau)$, but for $\tau = 0$ we will write $\delta(\sigma, 0)$. The Poisson equation (27) may be integrated directly with initial condition:

$$\left. \frac{\partial \varphi}{\partial \sigma} \right|_{\sigma=0} = const. \quad (29)$$

Because of the equation (27) structure *const* should be finite but it is not essential and it doesn't matter which the *const* is. The first integral of Poisson equation is:

$$\xi^2 \delta \frac{\partial \phi}{\partial \sigma} = 4\pi \gamma \mu(\gamma) \sigma - L(\tau) \quad (30)$$

where $L(\tau)$ is some function will be defined later from the conditions of independence all physical values from the axes of symmetry. Excluding the $\xi \delta \frac{\partial \varphi}{\partial \sigma}$ from (27) and (30) the equation:

$$\frac{\xi}{\kappa_0} \frac{\partial v}{\partial \tau} = -\frac{\xi^2}{\gamma} \frac{\partial P}{\partial \sigma} - \frac{4\pi^2 \gamma \mu(\gamma) \sigma - L(\tau)}{\gamma \mu(\gamma) M_0^2} \quad (31)$$

is obtained.

The following transformation is connected with the equation of continuity (26). Using the definition of velocity in the Lagrangian variables:

$$v = \frac{\partial \xi}{\partial \tau} \quad (32)$$

and dividing (26) by δ^2 the equation:

$$\frac{\partial}{\partial \tau} \frac{1}{\delta} = \frac{1}{\kappa_0} \frac{\partial^2 \xi^2}{\partial \tau \partial \sigma} \quad (33)$$

is obtained. Its integral is:

$$\frac{1}{\delta} - \frac{\kappa_0}{2} \frac{\partial \xi^2}{\partial \sigma} = G(\sigma) + \frac{1}{\delta(\sigma, 0)} - \frac{\kappa_0}{2} \frac{\partial \xi^2}{\partial \sigma} \Big|_{\tau=0} \quad (34)$$

The function $G(\sigma)$ will be defined from the initial conditions. From the definition of σ (24) and equality:

$$\frac{\partial \xi^2}{\partial \sigma} \Big|_{\tau=0} = \frac{d\xi^2}{d\sigma} \quad (35)$$

it follows that:

$$\frac{\partial \xi^2}{\partial \sigma} \Big|_{\tau=0} = \frac{2}{\delta(\sigma, 0)}. \quad (36)$$

Due to this

$$\frac{1}{\delta(\sigma, 0)} - \frac{\kappa_0}{2} \frac{\partial \xi^2}{\partial \sigma} \Big|_{\tau=0} = \frac{1 - \kappa_0}{\delta(\sigma, 0)}. \quad (37)$$

Finally, the integral of continuity equation is:

$$\frac{1}{\delta(\sigma, 0)} = \frac{\kappa_0}{2} \frac{\partial \xi^2}{\partial \sigma} + \frac{1 - \kappa_0}{\delta(\sigma, 0)} + G(\sigma) \quad (38)$$

Let's designate

$$\Sigma(\sigma) = \frac{1 - \kappa_0}{\delta(\sigma, 0)} + G(\sigma). \quad (39)$$

The next step is substitution the ρ from (38) into the equation of state (17) and motion equation (25):

$$\frac{\xi}{\kappa_0} \frac{\partial^2 \xi}{\partial \tau^2} = - \frac{\xi^2}{\gamma} \frac{\partial}{\partial \sigma} \left(\frac{\kappa_0}{2} \frac{\partial \xi^2}{\partial \sigma} + \Sigma(\sigma) \right)^{-\gamma} - \frac{4\pi^2 \gamma \mu(\gamma) \sigma - L(\tau)}{\gamma \mu(\gamma) M_0^2} \quad (40)$$

The function $L(\tau)$ is found out from the follows condition: all physical values are not dependent on the axes of cylinder. Let's use it for $\sigma = \xi = 0$. It is follows from (40):

$$L(\tau) = 0 \quad (41)$$

The function $G(\sigma)$ is found out from the initial conditions (28). To calculate it we introduce a parameter $\ddot{\xi}_0(\sigma)$:

$$\left. \frac{\partial^2 \xi}{\partial \tau^2} \right|_{(\sigma, 0)} = \ddot{\xi}_0(\sigma) \quad (42)$$

Using (36) - (39) and calculating (40) for $\tau = 0$ the equation for function $G(\sigma)$ is obtained:

$$G(\sigma)' - \frac{1}{\gamma} \left(G(\sigma) + \frac{1}{\delta} \right)^{\gamma+1} \left(\frac{\xi(\sigma, 0) \ddot{\xi}_0}{\kappa_0} - \frac{4\pi^2 \sigma}{M_0^2} \right) - \frac{\delta'}{\delta^2} = 0 \quad \text{where} \quad ' = \frac{d}{d\sigma}. \quad (43)$$

To find out the initial condition for this equation we calculate the (38) at point $\sigma = 0$ and obtain:

$$G(0) = 0. \quad (44)$$

So, the equation of radius motion which should be obtained in this section is:

$$\frac{\xi}{\kappa_0} \frac{\partial^2 \xi}{\partial \tau^2} = -\frac{\xi^2}{\gamma} \frac{\partial}{\partial \sigma} \left(\frac{\kappa_0}{2} \frac{\partial \xi^2}{\partial \sigma} + \Sigma(\sigma) \right)^{-\gamma} - \frac{4\pi^2 \sigma}{M_0^2} \quad (45)$$

with initial conditions

$$\xi|_{t=0} = \xi(\sigma, 0), \quad \left. \frac{\partial \xi}{\partial \sigma} \right|_{t=0} = \dot{\xi}(\sigma, 0) \quad (46)$$

where $v(\sigma, 0)$, $p(\sigma, 0)$, $\phi(\sigma, 0)$, $\delta(\sigma, 0)$ and $\xi(\sigma, 0)$ are specified functions. Function $G(\sigma)$ is defined by the equation (43) with initial condition (44). The equation (46) is nonlinear nonstationary partial differential equation which cannot be solved analytically in general but only after linierization.

6 The Linearization of Radiuse Motion Equation

The sections 5 and 6 are devoted to studying the small perturbations of the gas. An arbitrary radius motions are described by the equation (45) with initial conditions (46). According to the Model the transformation from (45) - (46) to linear approximation will be made now. Let's assume that a particle is in an equilibrium in the point of radius with Lagrange coordinat σ_0 . Because of smallness of deviation the particle from the point of equilibrium the Euler coordinat of the eparticle may be presented as a sum:

$$\xi(\sigma, \tau) = \xi_0 + \psi(\sigma, \tau) \quad \text{where} \quad |\psi(\sigma, \tau)| \ll \xi_0 \quad \text{and} \quad \xi_0 = \xi(\sigma_0, 0) \quad (47)$$

To study a generally case let assume that the particle is displaced at the moment of time $\tau = 0$ from the poin of equilibrium to new poin with lagrange coordinat

$\sigma = \sigma_0 + \Delta\sigma$ and has a velocity equal to $v(\sigma_0 + \Delta\sigma)$. So, the initial conditions according this model are

$$\xi(\sigma, 0) = \xi(\sigma_0, 0) + \psi(\sigma, 0) \quad \left. \frac{\partial \xi(\sigma, \tau)}{\partial \sigma} \right|_{\tau=0} = \dot{\psi}(\sigma, 0) \quad (48)$$

where

$$\psi(\sigma, 0) = \Psi \delta[\sigma - (\sigma_0 + \Delta\sigma)] \quad \dot{\psi}(\sigma, 0) = \dot{\Psi} \delta[\sigma - (\sigma_0 + \Delta\sigma)] \quad (49)$$

where Ψ and $\dot{\Psi}$ are parameters of the problem, and $\delta[\sigma - (\sigma_0 + \Delta\sigma)]$ is a delta-fuction. Let's designate:

$$\bar{\sigma} = \sigma + \Delta\sigma \quad (50)$$

In addition to (47) the condition

$$\left| \frac{\kappa_0}{2} \frac{\partial \xi^2}{\partial \sigma} \right| \ll \Sigma(\sigma) \quad (51)$$

will be used for linearisation of the equation (45). (51) allows to simplify

$$\left(\frac{\kappa_0}{2} \frac{\partial \xi^2}{\partial \sigma} + \Sigma(\sigma) \right)^{\gamma+1} \approx \Sigma(\sigma)^{\gamma+1} + (\gamma+1) \frac{\kappa_0}{2} \frac{\partial \xi^2}{\partial \sigma} \Sigma^\gamma(\sigma) \quad (52)$$

and after substitution the (52) into (45) the follwing equation is obtained:

$$\left(\frac{1}{\kappa_0 \xi} \frac{\partial^2 \xi}{\partial \tau^2} + \frac{4\pi^2}{M_0^2} \frac{\sigma}{\xi^2} \right) \left(\Sigma^{\gamma+1} + \kappa_0 \frac{\gamma+1}{2} \Sigma^\gamma \frac{\partial \xi^2}{\partial \sigma} \right) = \frac{\kappa_0}{2} \frac{\partial^2 \xi^2}{\partial \sigma^2} + \Sigma' \quad (53)$$

where $\Sigma' = \frac{d\Sigma}{d\sigma}$. To linearized (53) use the (47). This substitution gives the equaton

$$\frac{\partial^2 \psi}{\partial \tau^2} - \frac{\kappa_0^2 \xi_0^2}{\Sigma_0^{\gamma+1}} \frac{\partial^2 \psi}{\partial \sigma^2} + \frac{4\pi^2 \kappa_0^2 \gamma + 1}{\Sigma_0 M_0^2} \bar{\sigma} \frac{\partial \psi}{\partial \sigma} + \frac{4\pi^2 \kappa_0 \bar{\sigma}}{M_0^2 \xi_0} - \frac{\kappa_0 \xi_0 \Sigma_0'}{\Sigma_0^{\gamma+1}} = 0 \quad (54)$$

To simplify this equation denote

$$w_0^2 = \frac{\kappa_0^2 \xi_0^2}{\Sigma_0^{\gamma+1}} \quad B = \frac{4\pi^2 (\gamma+1) \kappa_0^2 \bar{\sigma}}{\Sigma_0 M_0^2} \quad C = \kappa_0 \xi_0 \frac{\Sigma_0'}{\Sigma_0^{\gamma+1}} - \frac{4\pi^2 \kappa_0 \bar{\sigma}}{M_0^2 \xi_0} \quad (55)$$

Then the equation (54) become as follows:

$$\frac{\partial^2 \psi}{\partial \tau^2} - w_0^2 \frac{\partial^2 \psi}{\partial \sigma^2} + B \frac{\partial \psi}{\partial \sigma} - C = 0 \quad (56)$$

with the initial conditions (49). B has a sense of acceleration. According to this two new characteristic values are introduced:

$$\sigma^* = \frac{w_0^2}{B} \quad \text{and} \quad \tau^* = \frac{\sigma^*}{w_0} \quad (57)$$

The equation (56) will be transformed now by the substitution

$$\psi(\sigma, \tau) = u(\sigma, \tau) \exp \frac{\sigma - \bar{\sigma}}{2\sigma^*} + C(\tau^*)^2 \frac{\sigma - \bar{\sigma}}{\sigma^*} \quad (58)$$

where $u(\sigma, \tau)$ is a new function. The result of the substitution is:

$$\frac{\partial^2 u}{\partial \tau^2} - w_0^2 \frac{\partial^2 u}{\partial \sigma^2} + \frac{u}{(2\sigma^*)^2} = 0 \quad (59)$$

For this equation the Cauchy problem is calculated with the following initial conditions which are obtained by transformation according to (58): begin:

$$u(\sigma, 0) = \psi(\sigma, 0) \quad \left. \frac{\partial u(\sigma, \tau)}{\partial \tau} \right|_{\tau=0} = \dot{\psi}(\sigma, 0) \quad (60)$$

It is the Klaine-Gordon equation ([3])

7 The Solution of the Klaine-Gordon Equation

To solve the Kline-Gordon equation the Riemann method of the Cauchy problem solution will be used ([4]). The equation (59) is a hyperbolic equation. In the canonic coordinates (ν, η) :

$$\nu = \frac{\sigma + w_0 \tau}{2\sigma^* w_0} \quad \eta = \frac{\sigma - w_0 \tau}{2\sigma^* w_0} \quad (61)$$

the equation (59) is reduced to the form:

$$\frac{\partial^2 u}{\partial \nu \partial \eta} - \frac{u}{4} = 0 \quad (62)$$

with initial conditions (60) transformed according to the rules: (61):

$$u(\sigma^* w_0(\nu + \eta))|_{\nu=\eta} = u(\sigma, 0) \quad \frac{1}{2\sigma^*} \left(\frac{\partial u}{\partial \nu} - \frac{\partial u}{\partial \eta} \right) \Big|_{\nu=\eta} = \frac{\partial u(\sigma, \tau)}{\partial \tau} \Big|_{\tau=0} \quad (63)$$

In the Riemann method the solution of the equation (62) is determined by the (63) and Riemann function $V(\nu, \eta)$ satisfying the conjugate equation:

$$\frac{\partial^2 V}{\partial \nu \partial \eta} - \frac{V}{4} = 0 \quad (64)$$

with initial conditions:

$$V(\nu, \hat{\eta}) = 1 \quad V(\hat{\nu}, \eta) = 1. \quad (65)$$

Let's find out the Riemann function V in the form

$$V = N(p) \quad \text{where} \quad p = \sqrt{(\nu - \hat{\nu})(\hat{\eta} - \eta)} \quad (66)$$

where (ν, μ) is the coordinate of the point where the solution found out and $(\hat{\nu}, \hat{\eta})$ belong to the line in which the initial conditions are given. From (67) it follows that $\tau = 0$ correspond to $\hat{\nu} = \hat{\eta}$. The substitution (67) into the (64) gives the ordinary differential equation:

$$N'' + \frac{1}{p}N' + N(p) = 0 \quad (67)$$

which has a finite solution:

$$N(p) = J_0 \left(\sqrt{(\nu - \hat{\nu})(\hat{\eta} - \eta)} \right) \quad (68)$$

where J_0 is a Bessel function of null order. The initial conditions (65) are satisfied. Let's return to the equation (62). According to the Riemann method the solution of the equation (62) satisfying the conditions (63) has a form:

$$\begin{aligned} u(\nu, \eta) = & \frac{1}{2} \left\{ u \left[\sigma^* w_0(\nu + \eta) \right]_{\nu=\eta} + u \left[\sigma^* w_0(\nu + \eta) \right]_{\eta=\nu} \right\} + \\ & \frac{1}{2} \int_{\eta}^{\nu} J_0 \left(\sqrt{(\nu - \hat{\nu})(\hat{\nu} - \eta_0)} \right) \left(\frac{\partial u}{\partial \nu} - \frac{\partial u}{\partial \eta} \right) \Big|_{\hat{\nu}=\hat{\eta}} d\hat{\nu} - \\ & \frac{\nu - \eta}{4} \int_{\eta}^{\nu} \frac{J_0' \left(\sqrt{(\nu - \hat{\nu})(\hat{\nu} - \eta)} \right)}{\sqrt{(\nu - \hat{\nu})(\hat{\nu} - \eta)}} u \left[\sigma^* w_0(\hat{\nu} + \hat{\eta}) \right]_{\hat{\nu}=\hat{\eta}} d\hat{\nu} \end{aligned} \quad (69)$$

Let's transform now the initial conditions (48) from the dependence on σ to dependence on $\hat{\nu}$:

$$\begin{aligned} u \left[\sigma^* w_0(\nu + \eta) \right]_{\nu=\eta} &= u(\sigma, 0) = \Psi \delta(\sigma - \sigma(0)) = \\ & \Psi \delta \left[\sigma^* w_0(\hat{\nu} + \hat{\eta}) \right]_{\hat{\nu}=\hat{\eta}} - \sigma(0) \\ \frac{1}{2\sigma^*} \left(\frac{\partial u}{\partial \nu} - \frac{\partial u}{\partial \eta} \right) \Big|_{\nu=\eta} &= \frac{\partial u(\sigma, \tau)}{\partial \tau} \Big|_{\tau=0} = \dot{\Psi} \delta(\sigma - \sigma(0)) = \\ & \dot{\Psi} \delta \left[\sigma^* w_0(\hat{\nu} + \hat{\eta}) \right]_{\hat{\nu}=\hat{\eta}} - \sigma(0) \end{aligned} \quad (70)$$

After substitution the (70) and

$$\nu - \eta = \frac{\tau}{\sigma^*} \quad (\nu - \hat{\nu})(\hat{\nu} - \eta) = (w_0 \tau)^2 - (\sigma - \sigma(0))^2 \quad (71)$$

into (69) the solution of the equation (62) with initial conditions (60) obtain:

$$u(\sigma, \tau) = \frac{1}{2} \{ \delta [w_0 \tau + (\sigma - \bar{\sigma})] + \delta [w_0 \tau - (\sigma - \bar{\sigma})] \} +$$

$$\frac{\dot{\Psi}\sigma(0)}{2w_0}J_0\left(\frac{\sqrt{(w_0\tau)^2 - (\sigma - \sigma(0))^2}}{2\sigma^*w_0}\right) - \frac{\tau\Psi w_0}{4}\frac{J_1\left(\frac{\sqrt{(w_0\tau)^2 - (\sigma - \sigma(0))^2}}{2\sigma^*w_0}\right)}{\sqrt{(w_0\tau)^2 - (\sigma - \sigma(0))^2}} \quad (72)$$

This formula together with (58) define the solution of linear problem.

8 Results and Discation

The obtained solution has a formal character of oscillations. But there are two reasons due to which this conclusion should be triated critically. First: This is a solution of a linear equation. The area of the initial conditions where linear solution differs slightly from the nonlinear one is not discuss in this article. The second: The solution has been obtained in the Lagrangian variables but when we are speaking about oscillation Euler variables are meant. The transformation from Lagrange variables to Euler sones is not performed in this article also.

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